

## Definitions.

The concept of empirical axiomatic theory is a formal representation of the “empirical content of data”. This concept originates from the logic of empirical theories [Pzelecki, 1969; Samokhvalov, 1973; Zagoruiko, Samokhvalov, et al, 1978] and from the critical analysis of representative measurement theory [Krantz et al, 1971, 1989, 1990].

**Definition.** Empirical axiomatic theory is a set  $M$  of four components:

$$M = \langle \text{Obs}^V, V, W, S \rangle, \text{ where}$$

$\text{Obs}^V$  is a measurement procedure,

$V = \{P_1, \dots, P_{n1}\}$  is a set of empirical predicates (we assume that the equality “=” belongs to  $V$ ),

$W = \{Q_1, \dots, Q_{n2}\}$  is a set of theoretical predicates, where the predicates from  $W$  are idealizations of the empirical predicates from  $V$ , and

$S$  - is a set of axioms in the  $V \cup W$ .

The set of axioms  $S$  consists of axioms  $S^V, S^W$  for  $V$  and  $W$  and for mapping rules  $S^{V \cup W}$ . These rules may be derived from the domain knowledge of measurement procedure  $\text{Obs}^V$  and predicates from  $V$ . If there is no mapping rule between  $V$  and  $W$ , then actually there is theoretical knowledge and sets  $W, S^W \cup S^{V \cup W}$  are empty. In this case, empirical axiomatic theory  $M$  consists of only three components:

$$M = \langle \text{Obs}^V, V, S^V \rangle.$$

Measurement procedure,  $\text{Obs}^V$ , interprets the empirical predicates from  $V$ . If this procedure is applied to a set of objects  $A = \{a_1, \dots, a_m\}$  then a formal protocol  $\text{pr}^V$  of observations is produced. This protocol includes symbols for objects  $a_1, \dots, a_m$ , symbols of predicates (from  $V$ ), and possibly some other symbols. It is assumed that measurement procedure  $\text{Obs}^V$  can be applied to any set of objects  $A$ . It can be done by introducing a third truth value (not defined) for the predicates from  $V$ . Next, for simplicity of consideration, it is assumed that  $\text{Obs}^V$  produces only one protocol of observations for a given  $A$ . Hence, the procedure  $\text{Obs}^V$  defines a mapping from the set of objects  $A$  to protocols:  $\text{Obs}^V(A) = \text{pr}^V$ .

The set of all formulas in the set  $V$ , which is true for all protocols of observation  $\text{pr}^V = \text{Obs}^V(A)$  is called empirical dependency.

We say that an empirical axiomatic theory has an empirical interpretation, if all parts of that theory are interpretable in the domain theory (background knowledge): measurement procedure  $\text{Obs}^V$ , protocol of observations  $\text{pr}^V$ , predicates from  $V$  and  $W$ , and axioms  $S$ .

The concept of empirical system can be defined in terms of empirical axiomatic system as a non-reducible model [Pfanzagl, 1971] of the set of axioms  $S^W$ . This means that the model does not merge objects, which are different for predicates from  $W$ .